7.5 Applications from Science and Statistics

Work Revisited

$W = F \cdot d$ $\quad dx \rightarrow$ distance moved

EXAMPLE 1 Finding the Work Done by a Force

Find the work done by the force $F(x) = \cos(\pi x)$ newtons along the x-axis from $x = 0$ meters to $x = 1/2$ meter.

$$\int_0^{1/2} (\cos \pi x)\, dx$$

$$-\frac{1}{\pi} \sin \pi x \Big|_0^{1/2}$$

$$= -\frac{1}{\pi}(1 - 0)$$

$$= -\frac{1}{\pi} \text{ Nm (Joules)}$$

EXAMPLE 2 Work Done Lifting

A leaky bucket weighs 22 newtons (N) empty. It is lifted from the ground at a constant rate to a point 20 m above the ground by a rope weighing 0.4 N/m. The bucket starts with 70 N (approximately 7.1 liters) of water, but it leaks at a constant rate and just finishes draining as the bucket reaches the top. Find the amount of work done

(a) lifting the bucket alone;

(b) lifting the water alone;

(c) lifting the rope alone;

(d) lifting the bucket, water, and rope together.

a) $W = Fd$ bucket alone

$= 22(20)$

$= 440$ J

b) $F = 70\left(-\frac{1}{20}x + 1\right)$

(0,1)
(20,0)

$= -3.5x + 70$

$\int_0^{20} (-3.5x + 70)\,dx$

$-\frac{3.5}{2}x^2 + 70x \Big|_0^{20}$ water

$-700 + 1400$

$= 700$ J

c) $F = .4(-x + 20)$

(0,20)
(20,0)

$\int_0^{20} (-.4x + 8)\,dx$ rope

$= -.2x^2 + 8x \Big|_0^{20}$

$= -80 + 160$

$= 80$ J

d) 440 J + 700 J + 80 J = 1220 J total